Int. J. Solids Structures Vol. 26, No. 1, pp. 73–82, 1990
Printed in Great Britain.

0020–7683/90 $3.00 + .00

AN ENGINEERING MULTIAXIAL CONSTITUTIVE MODEL FOR NONLINEAR TIME-DEPENDENT MATERIALS†

P. G. GLOCKNER
Department of Civil Engineering and Department of Mechanical Engineering, The University of Calgary, Calgary, Alberta, Canada

and

W. SZYSZKOWSKI
Department of Mechanical Engineering, The University of Saskatchewan, Saskatoon, Saskatchewan, Canada

(*Received* 16 *May* 1988; *in final form* 15 *March* 1989)

Abstract—A semi-empirical engineering constitutive law modelling in a unified and continuous manner the main characteristic features of time-dependent materials, including creep, strain softening, relaxation and recovery and tensile brittleness, is briefly reviewed. The model, which contains 13 parameters, is a hereditary single Volterra-type integral representation of material response with stress/strain nonlinearity assumed in the form of a power law, the strain tensor dependent on the entire stress history and the stress-anisotropy/brittleness feature handled by means of a tensile-stress dependent damage function. The capability/versatility of the model is illustrated by examples for several materials.

1. INTRODUCTION

Despite the general availability of computer codes and computing equipment in design offices and the recent progress in the application of principles of rational thermodynamics to constitutive theory, the analysis and design of structures and machines made of time-dependent materials continues to pose a serious challenge to the engineer. One of the stumbling blocks is the choice of an appropriate constitutive model which accurately and properly simulates the behaviour of real materials, a task made more difficult by the lack of sufficient and definitive experimental data concerning the time-dependent behaviour of common engineering and new/synthetic structural materials. Experimental evidence is particularly scarce with regard to multiaxial and brittle/fracture behaviour of these materials. A second contributing factor to the designer's difficulties is the recent emphasis on improved performance characteristics for structural materials, often under increasingly stringent/hostile environments, a demand which is largely the result of economic/competitive considerations, and which resulted in research and development work leading to completely new structural materials, such as fibre-reinforced plastics/resins and mechanical alloys or to significant improvements in the mechanical, chemical, thermal, electrical or transmissivity properties/responses of existing materials, examples of which are the "new and improved" ceramics.

Many researchers agree that a reasonably successful simulation of the spectrum of complex responses of these materials requires constitutive models of the hereditary type which take into account the entire stress/strain history of the material. Unfortunately, such models are rather complex, involving multiple Volterra-type integrals and a large number of material parameters/functions, to be determined experimentally (Sobotka, 1984). These models were tested/verified mostly against plastic materials (Ward, 1983).

In a traditional phenomenological approach, used in structural/solid/continuum mechanics and engineering design and also adopted here, the constitutive models basically describe the macroscopic behaviour and properties of the material without offering an explanation for or insight into the specific microscopic nature/mechanism of the phenom-

† The results presented here were obtained in the course of research sponsored by the Natural Sciences and Engineering Research Council of Canada, Grant No. A-2736.

enon modelled. In contradistinction to such an approach, there are models proposed by physicists, metallurgists and material scientists which are based on observations of deformation mechanisms at the microscopic level, using so-called internal state variables. The concept of a material state, borrowed from the theory of plasticity is used, usually using two (additional) state variables, namely the "drag-stress", defining the radius of the stress state surface, and the deviatoric "back-stress", locating the centre of this surface (Ramaswamy *et al.*, 1987). In proposing such models, referred to as "unified" constitutive models in the metallurgical/materials science literature, the proponents argue that since both creep (time-dependent) and plastic (time-independent) effects can be seen from a microstructural/metallurgical viewpoint as resulting from similar or identical mechanisms, these effects should be, and can be treated in a unified manner as "inelastic deformations" (Miller, 1987). Such a metallurgical approach is, however, limited to the modelling of metals. It also does not, to the best of the authors' knowledge, include/describe tertiary creep effects. Furthermore, such models require a very large number of material parameters/functions, the experimental determination of which may, and likely will represent quite a challenge. For example, some 30 parameters were suggested/specified in such a recent model (Slovik and Sehitoglu, 1987). Even if such parameters and functions could be defined/determined, there remains the non-trivial task of incorporating such a complex material law, together with appropriate equations of equilibrium and geometrical relations, into a set of governing equations which will be tractable and manageable with a reasonable computational effort. The difficulties encountered in achieving such a practical goal lead designers to revert to "simplified" models, their simplification being almost directly proportional to the complexity of the structure/analysis problem at hand (Kraus, 1980; Yagadir and Reddy, 1985). In analysis and numerical calculations, therefore, so-called simple creep theories are used in which heredity is neglected and in which "unsteady" (hardening) and "steady" (secondary) creep effects are usually defined by different sets of parameters and expressions, the latter often arrived at simply by curve-fitting of constant stress creep test data. The choice as to the type of creep law (expression) to be used in a particular case is left to the user who also has to decide when to switch from one law to another (Dietrich *et al.*, 1987). In addition to such practical difficulties, the use of simplified models gives predictions for material behaviour which very often do not compare favourably with corresponding experimental data and/or structural response, particularly where stresses are varying with time.

Some of these difficulties may be remedied by the introduction of a "unified" engineering constitutive model which simulates, in a continuous manner and by means of a single expression and a set of material parameters, the main characteristic features of time-dependent materials, including three stages of creep, relaxation and recovery, strain softening at constant strain rates an even tensile brittleness. It should be emphasized that our "unified engineering" model has nothing to do with the "unified constitutive theories" recently introduced by metallurgists and materials scientists using the "microscopic approach" mentioned above. Our model is intended to include metallic as well as non-metallic materials. We modified and relaxed the rigor of mathematical/continuum models so as to include engineering/analysis requirements (tractability and manageability) thereby, hopefully, arriving at a model which will be useful to the engineer and designer in his everyday activities. A material law of this type was developed and discussed in Szyszkowski *et al.* (1985) and Szyszkowski and Glockner (1985, 1986, 1987a), for materials with well-established microscopic structure (ageing excluded) and memory depending on this structure. The model is a hereditary single Volterra-type integral representation of material response with stress/strain nonlinearity assumed in the form of a power function (Norton's power law) and the strain rate tensor a function of the entire stress history. This (strain) rate tensor is decomposed into its elastic, viscous recoverable, and viscous permanent strain rate constituents. The model involves 13 material parameters, two standard elastic constants, five viscous parameters describing the first two stages of creep and three material constants for each of the tensile and compressive third stage of creep, parameters each one of which has specific physical interpretation/meaning and should be determinable from suitably designed experiments. Our model thus meets two primary objectives, namely:

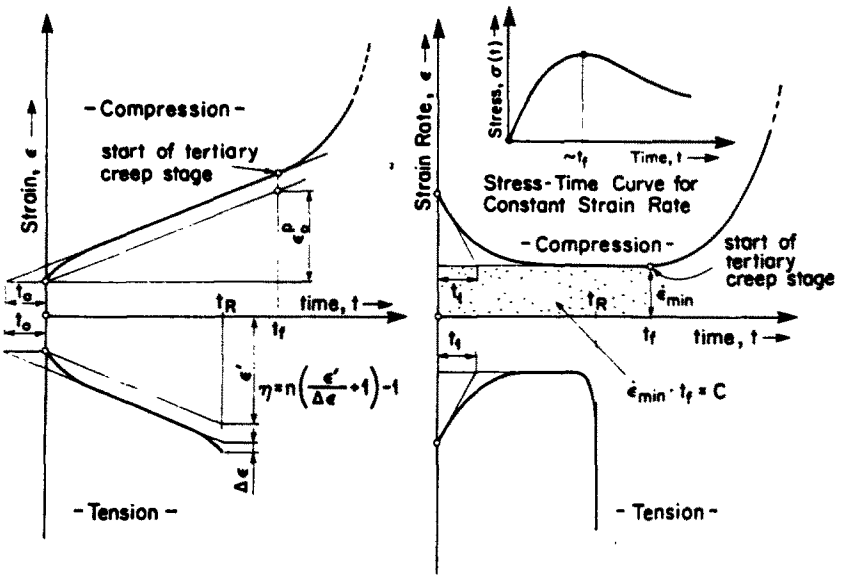

Fig. 1. Some characteristics features of tension and compression creep and compression constant strain rate test curves.

(i) it leads to a formulation which is "tractable" and "calculable" by means of some reasonable effort and
(ii) it contains only a "reasonable" number of material parameters which must be determinable from experiments.

In addition, based on experimental data, our model incorporates the following assumed material characteristics:

(1) a linear instantaneous response typical of Hookean bodies,
(2) material isotropy, extending for some range into the viscous deformation domain, including the primary and secondary creep stages,
(3) isotropy terminating with the appearance of microcracking at some advanced stage of deformation, a stage which can be related to the start of the accelerating/third phase of creep,
(4) the third creep stage for compression may be rather long while for tension this stage may disappear almost completely leading to very rapid fracture,
(5) the stress-anisotropy/brittleness feature is handled by a damage function assumed to be tensile stress dependent,
(6) microcracking affects only the permanent and not the reversible portion of the viscous strain rate,
(7) above a certain level of compressive strain, the strain rate depends not only on stress level but also on the current value of the accumulated permanent viscous strain.

Most of the material characteristics modelled are indicated schematically on Fig. 1 for uniaxial creep (constant stress) and constant strain-rate tests in tension and compression.

After a brief summary of the analytical aspects of the model, its capability and versatility are illustrated by comparing theoretical predictions with experimental data for:

(i) primary and secondary stage creep behaviour for ice, asbestos laminate, copper, asphalt, concrete and poly-vinyl-chloride,
(ii) recovery (upon unloading) for ice and asbestos laminate,
(iii) all three creep stages in compression and strain softening at constant strain rates for ice.

2. THE CONSTITUTIVE MODEL

A general uniaxial strain/stress relation for time-dependent materials at constant temperature is written as (Sobotka, 1984; Szyszkowski and Glockner, 1987a; Findley *et al.*, 1976; Krempl, 1975)

$$\varepsilon(t) = \int_0^t \hat{F}[\sigma(\tau), t]\, \mathrm{d}\tau \tag{1}$$

where ε and σ denote strain and stress, respectively, and $\hat{F}$ defines the viscoelastic properties of the material as a function of stress history, from $\tau = 0$ (the "virgin" state) to the current instant, $\tau = t$. It is assumed that the external loads are limited to magnitudes so as to keep the stress level below the yield stress and thus produce only an elastic instantaneous response. Based on uniaxial test data for materials to be modelled here, eqn (1) is rewritten in the form

$$\varepsilon(t) = \frac{\sigma(t)}{E_0} + \int_0^t F[\sigma(\tau)] \cdot L(t-\tau)\, \mathrm{d}\tau \tag{2}$$

in which the elastic and viscous responses are separated with E_0 denoting Young's modulus, F defines the stress nonlinearity while $L(t)$, a stress-independent material function, represents the "fading memory" of the material, assumed to be

$$L(t) = \frac{1}{v_1} j(t) + \frac{1}{v_2}; \quad \frac{\mathrm{d}L}{\mathrm{d}t} \leqslant 0 \tag{3}$$

with v_1 and v_2 being further material constants and the function $j(t)$ satisfying the condition $\mathrm{d}j/\mathrm{d}t < 0$, $j(t) \to 0$ for $t \to \infty$ and for convenience normalized as $j(0) = 1$. For a wide class of materials, an appropriate choice for the stress nonlinearity function is Norton's power law, also referred to as Glen's law, expressed as

$$F(\sigma) = \sigma \cdot |\sigma|^{n-1} \tag{4}$$

where n, a material constant, indicates viscous nonlinearity. All parameters associated with isotropic creep, namely n, v_1, v_2, and the function $j(t)$ are to be determined from creep test data for the first two stages (Szyszkowski and Glockner, 1987a). In fact, the function $j(t)$ does not need to be determined explicitly. Instead, the parameters t_0 and t_1 (see Fig. 1) can be used to define an approximate creep characteristic, $\tilde{j}(t)$, such that

$$\int_0^\infty j(t)\, \mathrm{d}t = \int_0^\infty \tilde{j}(t)\, \mathrm{d}t; \quad \text{and} \quad \left.\frac{\mathrm{d}j}{\mathrm{d}t}\right|_{t=0} = \left.\frac{\mathrm{d}\tilde{j}}{\mathrm{d}t}\right|_{t=0};$$

in which the function $\tilde{j}(t)$, in turn, describes the creep behaviour of a generalized Kelvin body, also discussed in Szyszkowski and Glockner (1987a).

Using eqns (3) and (4) in eqn (2), the total strain, or the strain rate can be expressed in terms of its elastic, $\dot{\varepsilon}^{\mathrm{e}}$, reversible, $\dot{\varepsilon}^{\mathrm{r}}$, and permanent, $\dot{\varepsilon}^{\mathrm{p}}$, portions as

$$\dot{\varepsilon}(t) = \dot{\varepsilon}^{\mathrm{e}} + \dot{\varepsilon}^{\mathrm{r}} + \dot{\varepsilon}^{\mathrm{p}} = \frac{\dot{\sigma}(t)}{E_0} + \frac{1}{v_1}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t [\sigma(\tau)]^n \cdot j(t-\tau)\, \mathrm{d}\tau + \frac{1}{v_2}[\sigma(t)]^n \tag{5}$$

an expression which can formally be obtained by postulating the existence of a complementary power potential $\mathscr{P}(\sigma)$ in the form

$$\mathscr{P}(\sigma) = \int_0^{\sigma} \dot{\varepsilon}\, \mathrm{d}\sigma = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\sigma^2}{2E_0}\right] + \frac{1}{v_1}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \frac{[\sigma(\tau)]^{n+1}}{(n+1)} * j(t-\tau)\, \mathrm{d}\tau + \frac{1}{v_2}\frac{[\sigma(t)]^{n+1}}{n+1} \tag{6}$$

and writing $\dot{\varepsilon} = \mathrm{d}\mathscr{P}/\mathrm{d}\sigma$. Analogously, we define a complementary power potential, $\mathscr{P}(\sigma_{ij})$, for the three-dimensional stress state as

$$\mathscr{P}(\sigma_{ij}) = \int_0^{\sigma_{ij}} \varepsilon_{ij}\, \mathrm{d}\sigma_{ij} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\sigma_{kk}^2}{6K_0} + \frac{S^2}{6G}\right] + \frac{1}{v_1}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \frac{S^{n+1}}{(n+1)} * j(t-\tau)\, \mathrm{d}\tau + \frac{1}{v_2}\frac{S^{n+1}}{(n+1)} \tag{7}$$

from which we obtain

$$\dot{\varepsilon}_{ij} = \frac{\partial \mathscr{P}}{\partial \sigma_{ij}} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{r} + \dot{\varepsilon}_{ij}^{p} \tag{8}$$

and where

$$\dot{\varepsilon}_{ij}^{e} = \frac{1}{E_0}[(1+\mu)\dot{\sigma}_{ij} - \mu\dot{\sigma}_{kk}\delta_{ij}]; \quad \dot{\varepsilon}_{ij}^{r} = \frac{1}{v_1}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t [\tilde{\sigma}_{ij}(\tau)]^n j(t-\tau)\, \mathrm{d}\tau \tag{9a, b}$$

$$\dot{\varepsilon}_{ij}^{p} = \frac{1}{v_2}[\tilde{\sigma}_{ij}(t)]^n; \quad \tilde{\sigma}_{ij} = s_{ij}\left[\frac{3}{2}\left|\frac{S}{s_{ij}}\right|^{n-1}\right]^{1/n} \tag{9c, d}$$

in which $\tilde{\sigma}_{ij}$ denotes the "effective" viscous stress and $S^2 = \frac{1}{2}s_{ij}s_{ij}$ with the stress deviator defined in the normal manner by $s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$, μ denotes Poisson's ratio, K_0 and G are the bulk and shear modulus, respectively, while δ_{ij} are components of the Kronecker delta. Note that the viscous portions of $\mathscr{P}(\sigma_{ij})$ were assumed to depend only on the second stress deviator invariant, S, and therefore the viscous strain rates refer to incompressible viscous processes, neglecting the effects of the first stress invariant (the mean stress) on the viscous deformations, an assumption which is valid except for conditions of unusually high all-around (hydrostatic) pressures. The constitutive law in the form given by eqn (9) was discussed in Szyszkowski and Glockner (1987a), where a numerical method for the effective treatment of the integrals in the reversible portion of the creep was also proposed. Clearly, this law can describe only isotropic materials. There are, however, many time-dependent materials which, though their elastic or even primary viscous behaviour can be considered isotropic, may develop some stress anisotropy (despite their structural isotropy) which manifests itself in different responses to prolonged tension and compression. This anisotropy can be related to internal microcrackings. Resistance of the material with microcrackings is usually different for tensile and compressive stresses.

In accordance with one of the assumed material characteristics, microcracking affects only the permanent portion of the viscous strain rate, $\dot{\varepsilon}_{ij}^{p}$, an assumption based on experimental data indicating that at advanced stages of creep, the rate of reversible viscous strain, $\dot{\varepsilon}_{ij}^{r}$, approaches zero. Thus, only the expression for $\dot{\varepsilon}_{ij}^{p}$, eqn (9c), has to be modified for the tertiary phase, such modifications being different for compression and tension for our assumed "tensile–brittle" material. Note that this constitutive law, eqn (8), reduces to the well-known relations of linear viscoelasticity (Sobotka, 1984; Rabotnov, 1969), when we set $n = 1$.

2.1. *Microcracking in tension*

In a tensile zone, the effects of microcracking may be represented, qualitatively and in a global way, by introducing a damage function, $\omega(t)$, which physically describes the relative decrease in effective area available for stress transmission. The actual or "true" stress in such a zone, which is the stress level that governs the viscous process, is thus obtained by dividing the "nominal" stress by $1-\omega(t)$, with $\omega(t)$ varying from 0 for an uncracked (virgin)

material to 1 at the instant of rupture. Accordingly, the expression for the permanent viscous strain rate, eqn (9c), is modified to read

$$\dot{\varepsilon}_{ij}^{p} = \frac{1}{\nu_2} \cdot \left[\frac{\tilde{\sigma}_{ij}(t)}{1-\omega(t)} \right]^n. \tag{10}$$

To define the evolution law for the damage function we assume the rate of damage to be a function of the true stress and the damage itself, in the form

$$\dot{\omega}(t) = K \cdot \left[\frac{\sigma_{max}}{1-\omega(t)} \right]^{\Gamma} \cdot \frac{1}{[1-\omega(t)]^{\eta-\Gamma}} \tag{11}$$

where K, Γ and η are material constants determinable from and defining the tertiary (tensile) creep stage of the material, and σ_{max} denotes the largest tensile stress, assumed to be zero in the case of compression. The parameter η represents the ducility of the material near/at brittle failure; thus by choosing $\eta < \Gamma$ (or $\eta > \Gamma$) one may expand (contract) the tertiary phase for creep tests in tension (see Szyszkowski and Glockner, 1986). Setting $\sigma_{max} = \sigma_0 = \text{const.}$ in eqn (11), one can integrate the expression to obtain a relation involving the rupture time, t_R (for which $\omega = 1$), in the form

$$(1+\eta) K \cdot t_R \sigma_0^{\Gamma} = 1 \tag{12}$$

from which one can determine K and Γ by using a log–log plot of t_R vs σ_{max}.

2.2. *Microcracking in compression*

Effects of microcracking are not as severe in compression as in tension with the tertiary compressive creep stage for most materials under consideration here being significantly longer than the corresponding phase for tension. Strain levels during this stage reach magnitudes much larger than failure strains in tension. Experimental evidence for certain materials, e.g. ice, indicates that the onset of the tertiary (compressive) creep stage is associated with a certain level of "accumulated" strain and is independent of stress magnitude. The start of this phase, which is considered to contribute to and be primarily responsible for the development of microcracking, is at the instant, t_f (the "failure" time), which appears to be related to the minimum strain rate, $\dot{\varepsilon}_{min}$, by

$$\dot{\varepsilon}_{min} \cdot t_f = C \tag{13}$$

where C is a material constant (for ice: $C \cong 0.7$–1.0%).

In view of the strain decomposition used in eqn (8), the parameter C denotes the accumulated permanent viscous strain at time t_f. This limiting value of the permanent viscous strain, $\varepsilon_0^p = C$, is taken to be a material parameter also for the multiaxial stress state, where it should be measured by the "effective" axial permanent strain given by (Szyszkowski and Glockner, 1986)

$$\varepsilon_{ef}^{p} = [\tfrac{2}{3}\varepsilon_{ij}^{p}\varepsilon_{ij}^{p}]^{1/2}. \tag{14}$$

Thus, the term correcting the permanent viscous strain rate, eqn (9c), taking into account the effects of microcracking for the compression case, is written as

$$1+\alpha\left(\frac{\varepsilon_{ef}^{p}}{\varepsilon_0^p}-1\right)^a; \quad \text{where} \quad \alpha = 0 \quad \text{for} \quad \varepsilon_{ef}^{p} < \varepsilon_0^p \tag{15}$$

and where α and a, together with ε_0^p, are the material parameters to be determined from and defining the tertiary compressive creep phase of the material, up to failure.

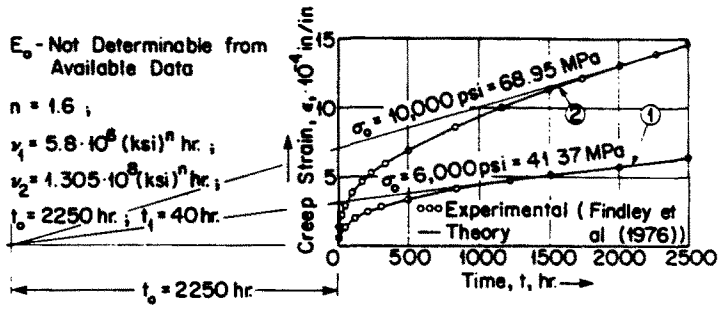

Fig. 2. Creep curves for oxygen-free copper in tension at 329°F (165°C).

The following procedure is suggested for modelling the compression creep behaviour of a material up to failure:

(i) for $\varepsilon^p_{ef} < \varepsilon^p_0$, microcracking does not affect the permanent viscous creep rate, $\dot{\varepsilon}^p_{ij}$, eqn (9c),

(ii) for $\varepsilon^p_{ef} \geqslant \varepsilon^p_0$, the permanent viscous creep rate is a function not only of the stress magnitude but also of the "excess" permanent viscous strain, $\varepsilon^p_{ef} - \varepsilon^p_0$.

While in the tension case, eqn (12) is used to define the instant of rupture, t_R, as a function of stress magnitude, the onset of failure in the compression case is identified from eqn (13) as the time, t_f, representing the start of the accelerating creep stage. Using the relation between $\dot{\varepsilon}_{min}$ and the stress magnitude, which according to eqn (9c) should have the form

$$\dot{\varepsilon}_{min} = \frac{1}{\nu_2}\sigma^n, \tag{16}$$

expression (13) can be recast as

$$t_f \cdot \sigma^n = \nu_2 C \tag{17}$$

which, like eqn (12), represents a straight line on a log–log plot of σ vs t_f.

Combining the contributions for the tensile and compressive tertiary creep stage, eqns (10) and (15), the final "corrected" expression for the permanent viscous strain rate for an arbitrary (positive or negative) stress state, taking into account the effects of microcracking, is written as

$$\dot{\varepsilon}^p_{ij} = \frac{1}{\nu_2}\left[\frac{\tilde{\sigma}_{ij}(t)}{1-\omega(t)}\right]^n \cdot \left[1+\alpha\left(\frac{\varepsilon^p_{ef}}{\varepsilon^p_0}-1\right)^a\right]; \quad \alpha = 0 \quad \text{for} \quad \varepsilon^p_{ef} < \varepsilon^p_0. \tag{18}$$

The nonlinear multiaxial constitutive model, represented by eqns (8), (9) and (18), reduces to the uniaxial nonlinear material law discussed in Szyszkowski and Glockner (1985), where it was also verified against experimental data from compression experiments on ice, both constant stress (creep) and constant strain rate tests.

3. CAPABILITY AND VERSATILITY OF MODEL

Due to the limited experimental data available, the model's capability in predicting/simulating the behaviour of a number of real materials could only be assessed for the uniaxial case. Using material parameters as indicated on the figures, the (uniaxial) primary and secondary stage creep response is predicted and compared with experimental data for oxygen-free copper in tension, asphalt concrete in compression and poly-vinyl-chloride in tension (Figs 2–5). The material parameters were evaluated by fitting the curves obtainable from eqn (5) to the available experimental data. The agreement between test

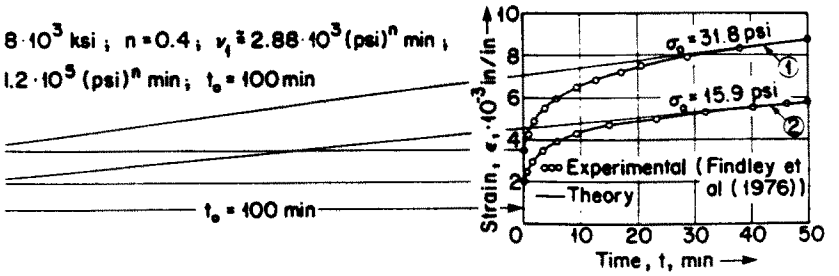

Fig. 3. Creep response of asphalt concrete in compression at room temperature.

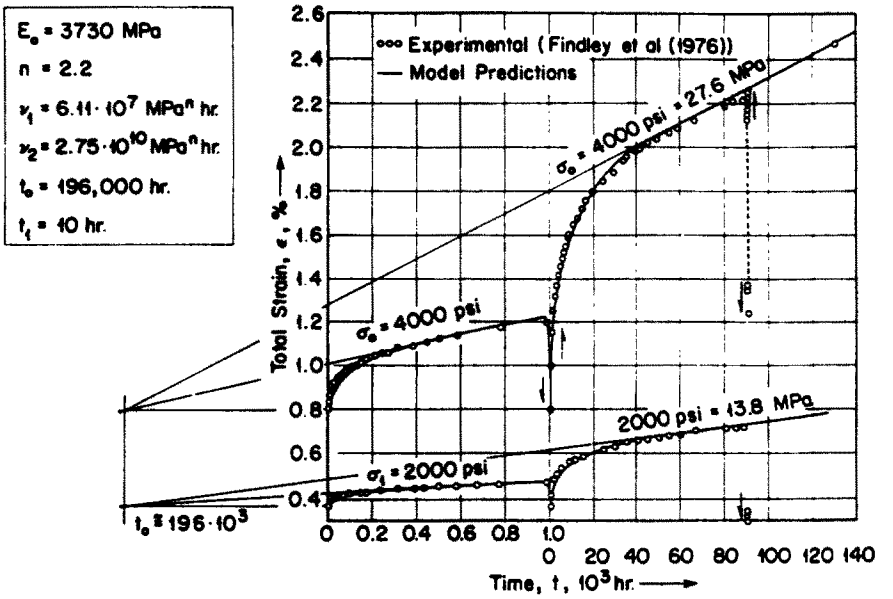

Fig. 4. Model predictions and experimental results for long-time creep of poly-vinyl-chloride.

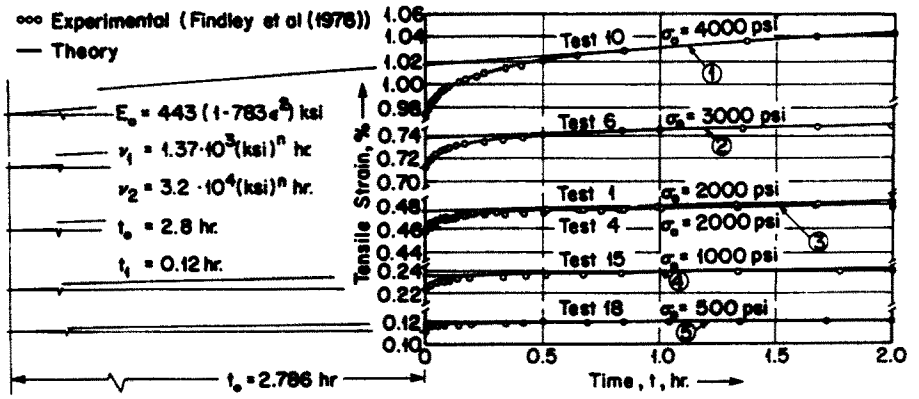

Fig. 5. Creep curves of unplasticized poly-vinyl-chloride in tension.

results and theoretical prediction verifies, to some extent, the validity of the creep model discussed in the previous chapter.

Theoretical/model results for primary and secondary creep as well as recovery after unloading are compared with experimental data for asbestos laminate and for ice next (Figs 6 and 7). Note the excellent agreement between predictions and test data for the creep response of both materials and the recovery phase of ice while the recovery for asbestos laminate is clearly overestimated, the model not taking into account permanent set due to plastic effects which are likely to be present in such materials.

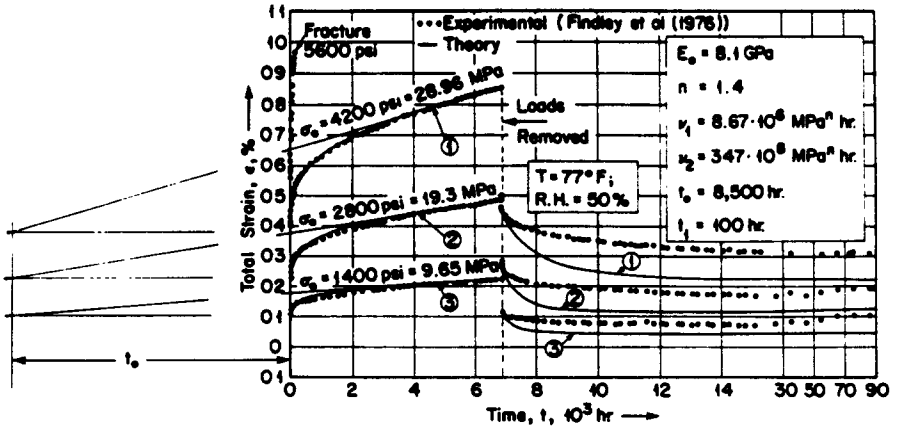

Fig. 6. Comparison of experimental data and model predictions for creep and recovery of asbestos laminate.

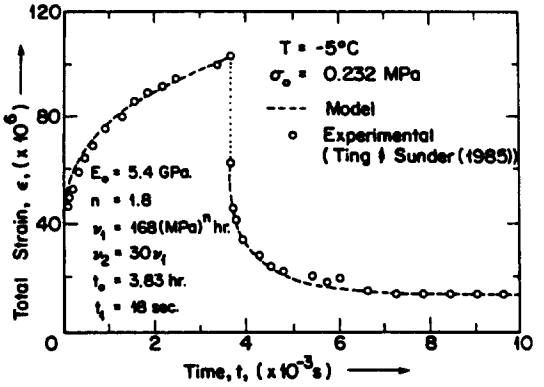

Fig. 7. Creep and recovery data for ice.

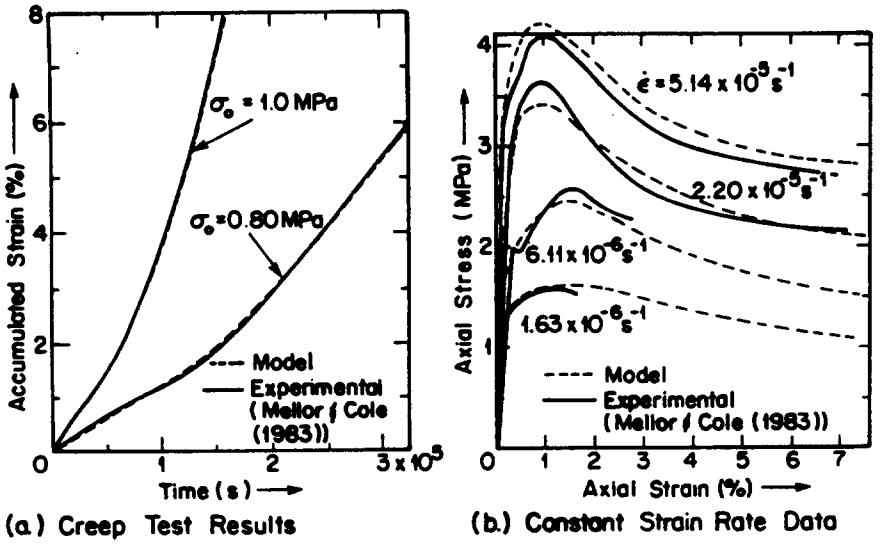

Fig. 8. Comparison of model predictions with experimental data for ice.

The final figure (Fig. 8) shows the capability of the model in predicting all three stages of creep and strain softening at various constant strain rates for ice, again simulating actual material behaviour with a surprising precision (Szyszkowski and Glockner, 1985). The creep curves and constant strain-rate response are obtained, of course, using one and the same set of material parameters (Szyszkowski and Glockner, 1985).

4. CONCLUSIONS

A multiaxial hereditary constitutive model for nonlinear time-dependent materials is briefly reviewed and its capability and versatility demonstrated by predicting the uniaxial behaviour in tension or compression of a number of materials and comparing such predictions with experimental data. The model describes, in a unified and continuous manner and by means of a single expression and a set of material parameters, the three stages of creep, relaxation and recovery, strain softening at constant strain rates and tensile brittleness. It takes into account the entire stress history in defining the strain/stress relation, and treats arbitrary spatial and temporal variation in stress, including the case when $\sigma = 0$ (after unloading), after which there is some strain recovery; for many "simplified" models in use, $\sigma = 0$ means $\dot{\varepsilon} = 0$, with no recovery predicted. Only one material function, the stress-nonlinearity $F(\sigma) = \sigma^n$, is used while all other parameters are material constants independent of stress level. It is significant that the model can predict, with apparently surprising accuracy, the response of a fairly wide class of materials under a variety of stress/strain conditions, both compressive and tensile, using a single set of material parameters. In this sense, our model is also superior to most of the commonly used, "simple" creep models which normally use a large number of parameters/functions, many of which are functions of stress level (Dietrich *et al.*, 1987). Additionally, the hereditary model discussed here can easily be adopted to any FEM program (Szyszkowski and Glockner, 1987b) for numerical analysis of complex structures.

Unfortunately, due to lack of multiaxial experimental data, only the uniaxial version of the model could be applied and assessed. In addition, such evaluation was restricted primarily to the first two stages of creep and recovery phenomena, with tertiary creep and strain softening data also being extremely scarce, particularly for the tensile stress state.

Despite its capabilities and its versatility, this semi-empirical model should be looked upon as a first step towards finding a general constitutive law for this class of materials. Substantial progress in reaching such a goal will require considerable additional experimental data, including results from tensile and multiaxial tests.

REFERENCES

Dietrich, D. E., Power, M. R. and Finkel J. I. (Eds) (1987). Panel discussion in *Proc. ANSYS* 1987 *Conf. on Integrating Design and Analysis*, Newport Beach, California.

Findley, W. N., Lai, J. S. and Onaran, K. (1976). *Creep and Relaxation of Nonlinear Viscoelastic Materials*. North-Holland, Amsterdam.

Kraus, H. (1980). *Creep Analysis*. John Wiley, New York.

Krempl, E. (1975). On the interaction of rate and history dependence in structural metals. *Acta Mech.* **22**, 53–90.

Mellor, M. and Cole, D. M. (1983). Stress/strain/time relations for ice under uniaxial compression. *Cold Regions Sci. Tech.* **6**, 207–230.

Miller, A. K. (Ed.) (1987). *Unified Constitutive Equations for Creep and Plasticity*. Elsevier, Amsterdam.

Rabotnov, Yu. N. (1969). *Creep Problems in Structural Members*. North-Holland, Amsterdam.

Ramaswamy, V. G., Stouffer, D. C., Van Stone, R. H. and Laften, J. H. (1987). Modelling thermomechanical cyclic response with a "unified" state variable constitutive equation. In *Thermal Stress, Material Deformation and Thermo-Mechanical Fatigue. ASME-PVP* **123**, 57–65.

Slovik, D. and Sehitoglu, H. (1987). A constitutive model for high temperature loadings, I and II. In *Thermal Stress, Material Deformation and Thermo-Mechanical Fatigue. ASME-PVP* **123**, 65–83.

Sobotka, Z. (1984). *Rheology of Materials and Engineering Structures*. Elsevier, Amsterdam.

Szyszkowski, W., Dost, S. and Glockner, P. G. (1985). A nonlinear constitutive model for ice. *Int. J. Solids Structures* **21**, 307–321.

Szyszkowski, W. and Glockner, P. G. (1985). Modelling the time-dependent behaviour of ice. *Cold Regions Sci. Tech.* **11**, 3–21.

Szyszkowski. W. and Glockner, P. G. (1986) On a multiaxial constitutive law for ice. *Mech. Materials* **5**, 49–71.

Szyszkowski, W. and Glockner, P. G. (1987a). On a multiaxial non-linear hereditary constitutive law for non-ageing materials with fading memory. *Int. J. Solids Structures* **23**, 305–324.

Szyszkowski, W. and Glockner, P. G. (1987b). On hereditary effects in creep computations. *Proc. ANSYS Conf. on Integrating Design and Analysis*, Swanson Analysis Systems, Inc., 9.56–9.66.

Ting, S. K. and Sunder, S. S. (1985). Constitutive modelling of sea ice with applications to indentation problems. MIT CSEOE Report No. 3.

Ward, I. M. (1983). *Mechanical Properties of Solid Polymers*. John Wiley, New York.

Yagadir, S. and Reddy, C. P. (1985). Viscoelastic analysis of nearly incompressible solids. *Comput. Struct.* **20**, 235–243.